Math 60 8.5 Linear Functions and Models (Day 2)

Objectives: Build linear models from verbal descriptions

Build linear models from data

In particular

4) Cost functions
5) Straight-line depreciation
6) Models when two data points given

(1) Tony's weekly salary selling cars is 0.75% of his weekly sales plus $450 so his salary each week is given by

$$S(x) = 0.0075x + 450$$

where x is the total value of the cars he sells ("his sales") in dollars.

a) What is the domain implied by this function?

Sales cannot be negative, so

$x \geq 0$, giving domain $\boxed{[0, \infty)}$

b) If Tony sells cars worth $50,000 one week, what is his salary?

$x = 50,000$

find $S(50,000) = 0.0075(50000) + 450$

$= \boxed{\$825}$ * Always include units *

c) If Tony earned $840 one week, what was the value of the cars he sold?

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$S(x) = 0.0075x + 450$

Substitute $S(x) = 840$

$840 = 0.0075x + 450$ Solve for x

$840 - 450 = 0.0075x$

$\frac{390}{0.0075} = \frac{0.0075x}{0.0075}$

$\boxed{\$52{,}000 = x}$ * Always include units!

② The weekly cost R of renting an RV is #129 plus #0.32 per mile, up to a maximum of 500 miles.

a) Find a linear function that expresses R as a function of miles, m.

This means find $R(m)$.
We will treat $R \leftrightarrow y$ analog.
and $m \leftrightarrow x$ analog.

Units of R are #.
Units of m are miles. } Use the units to understand the question!

The number \$129 has units with #, so it is a value of R.

It costs \$129 if we drive no miles ($m=0$), so $(0, 129)$ is a point on our graph and 129 is the y-intercept (or R-intercept).

In "\$0.32 per mile", "per" means divide. $\frac{\$0.32}{\text{mile}}$

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Remember that slope $= \frac{y_2 - y_1}{x_2 - x_1}$

which is $\frac{R_2 - R_1}{m_2 - m_1}$ or $\frac{\$ - \$}{mi - mi} = \frac{\$}{mi}$

CAUTION
Let's not call slope m because our problem uses m to mean miles, the x-variable

We don't need to calculate slope, but we use the "per" word to help us identify that slope = 0.32

So we know slope = 0.32
y-int = 129

So using $y = mx + b$
$R(m) = \text{slope} \cdot m + R\text{-int}$
$\boxed{R(m) = 0.32m + 129}$ or $\boxed{R = 0.32m + 129}$

b) What are the independent and dependent variables?

x is usually independent → $\boxed{m \text{ is independent}}$
y is usually dependent → $\boxed{R \text{ is dependent}}$

c) What is the domain of this linear function?

We cannot go negative miles so $m \geq 0$
But the cost structure applies for 500 miles
so $m \leq 500$

Putting these together $0 \leq m \leq 500$
interval $\boxed{[0, 500]}$

d) What is the rental cost if 360 miles are driven?

$R(360) = 0.32(360) + 129$
$= \boxed{\$244.20}$

e) How many miles were driven if the rental cost was \$275.56?

cont →

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$R(m) = 0.32m + 129$

Substitute $R(m) = 275.56$

$275.56 = 0.32m + 129$

Solve for m

$275.56 - 129 = 0.32m$

$$\frac{146.56}{0.32} = \frac{0.32m}{0.32}$$

$\boxed{m = 458 \text{ miles}}$

(3) According to the National Center for Health Statistics, the average birth weight of babies born to 22-year-old mothers is 3280 g. The average birth weight of babies born to 32-year-old mothers is 3370 grams. Suppose that the relation between age of the mother and birth weight is linear.

a) Find a linear function that relates age of mother, a, to birth weight w, treating age of mother as the independent variable.

(a, w) ordered pairs (independent, dependent)

means we have two points

(22, 3280) and (32, 3370)

Find slope: $\frac{y_2 - y_1}{x_2 - x_1}$ becomes $\frac{w_2 - w_1}{a_2 - a_1}$ $\frac{\text{dependent}}{\text{independent}}$

$$\frac{3370 - 3280}{32 - 22} = \frac{90}{10} = 9$$

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Substitute into point-slope formula

$y - y_1 = m(x - x_1)$ or $w - w_1 = m(a - a_1)$

$y - 3280 = 9(x - 22)$

Hint
Most students find it easier to work out using x and y.

distribute

$y - 3280 = 9x - 198$

$y = 9x - 198 + 3280$ isolate y

$y = 9x + 3082$

change to variables specified by the problem

$\boxed{w(a) = 9a + 3082}$

b) Predict the birthweight of a baby born to a mother who is 30 years old.

Find $w(30) = 9(30) + 3082$

$= \boxed{3352 \text{ grams}}$

c) If a baby's birth weight is 3310 grams, how old do we expect the mother to be?

Substitute 3310 for $w(a)$

$3310 = 9a + 3082$

Solve for a

$3310 - 3082 = 9a$

$\frac{228}{9} = \frac{9a}{9}$

$a = 25\frac{1}{3}$ years

Call her $\boxed{\text{age 25 years}}$

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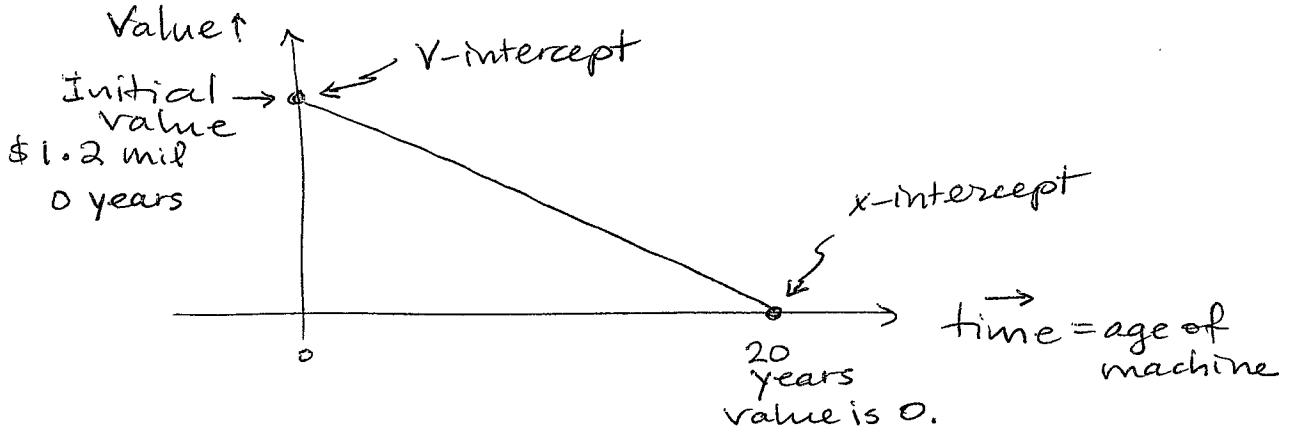
(4) Suppose that a company just purchased a new machine for its manufacturing facility for $1,200,000. The company chooses to depreciate the machine using the straight-line method over 20 years.

Depreciation refers to <u>loss of value</u> and is used in tax calculations and in accounting for the current value of equipment.
The total time given in the question is the amount of time until the equipment value is 0.



a) Find a linear function that expresses the value V of the machine as a function x of its age

(0, 1200000) and (20, 0) are two points on the graph of the value

Find slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\left[\frac{V_2 - V_1}{x_2 - x_1}\right.$ in the notation of this question$\left.\right]$

$$= \frac{1200000 - 0}{0 - 20}$$

$$= \frac{1200000}{-20}$$

$$= -60000$$

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In depreciation problems, the y-intercept is the initial value.

$(0, 1200000)$

Substitute into slope-intercept form:

$$y(x) = -60000x + 1200000$$

Change to variables specified

$$\boxed{V(x) = -60000x + 1200000}$$

b) What is the domain

$0 \leq$ time ≤ 20 years

$$\boxed{[0, 20]}$$

c) What is the book value of the machine after 3 years

$x = 3$

Find $V(3) = -60000(3) + 1200000$

$$= \boxed{\$1,020,000}$$

d) When will the book value be \$480,000

Replace $V(x)$ by 480 000:

$$480\,000 = -60000x + 1200000$$

$$480\,000 - 1200000 = -60000x$$

$$\frac{-720000}{-60000} = \frac{-60000x}{-60000}$$

$$\boxed{12 \text{ years} = x}$$

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Objectives, continued:

4) Cost functions
5) Straight-line depreciation
6) Models where two data points are given

Examples

1) Tony's weekly salary selling cars is 0.75% of his weekly sales, plus \$450, so his salary each week is given by $S(x) = 0.0075x + 450$, where x is the total value of the cars he sells ("his sales"), in dollars.
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   b. If Tony sells cars worth \$50,000 one week, what is his salary that week?

   c. If Tony earned \$840 one week, what was the value of the cars he sold?

2) The weekly cost R of renting an RV is \$129 plus \$0.32 per mile, up to a maximum of 500 miles.
   a. Find a linear function that expresses R as a function of miles, m.

   b. What are the independent and dependent variables?

   c. What is the domain of this linear function?

   d. What is the rental cost if 360 miles are driven?

   e. How many miles were driven if the rental costs was \$275.56?

3) According to the National Center for Health Statistics, the average birth weight of babies born to 22-year-old mothers is 3280 grams and the average birth weight of babies born to 32-year-old mothers is 3370 grams. Suppose that the relation between age of the mother and birth weight is linear.

a. Find a linear function that relates age of mother, a, to birth weight, w, treating age of mother as the independent variable.

b. Predict the birth weight of a baby born to a mother who is 30 years old.

c. If a baby's birth weight is 3310 grams, how old do we expect the mother to be?

4) Suppose that a company just purchased a new machine for its manufacturing facility for $1,2000,000. The company chooses to depreciate the machine using the straight-line method, over 20 years.

a. Find a linear function that expressed the value V of the machine as a function of x, its age.

b. What is the domain of this function?

c. What is the book value of the machine after 3 years?

d. When will the book value be $480,000?